

# Modulation stabilization of Bloch oscillations of two-component Bose-Einstein condensates in optical lattices

Huai-Qiang Gu<sup>1</sup> ‡, Jun-Hong An<sup>2</sup> and Kang Jin<sup>3</sup>

<sup>1</sup>School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China

<sup>2</sup>Center for Interdisciplinary Studies, Lanzhou University, Lanzhou 730000, China

<sup>3</sup>Department of Physics, Northwest University, Xi'an 710118, China

**Abstract.** We study the Bloch oscillations (BOs) of two-component Bose-Einstein condensates (BECs) trapped in spin-dependent optical lattices. Based on the derived equations of motion of the wave packet in the basis of localized wave functions of the lattice sites, the damping effect induced by the intercomponent and intracomponent interactions to the BOs is explored analytically and numerically. We also show that such damping of the BOs can be suppressed entirely if all the atom-atom interactions are modulated synchronously and harmonically in time with suitable frequency via the Feshbach resonance. When the intercomponent and the intracomponent interactions have inverse signs, we find that the long-living BOs and even the revival of the BOs can be achieved via only statically modulating the configuration of optical lattices. The results provide a valuable guidance for achieving long-living BOs in the two-component BEC system by the Feshbach resonances and manipulating the configuration of the optical lattices.

PACS numbers: 67.85.Hj, 67.85.Fg, 67.85.De, 52.35.Mw

‡ Email: guhq06@lzu.edu.cn

## 1. Introduction

The system of ultracold atomic gases in optical lattices has become a nice experimental platform to simulate the effects in condensate matter physics [1]. The well controllability of such system makes many sophisticated effects in condensed matter physics be well studied in this system [2]. The Bloch oscillation (BO) is the oscillatory motion of a quantum particle in a periodic potential when it is subjected to an external force. It was originally predicted in solid-state system where the motions of the electrons in tilted periodic potentials undergo coherent oscillations [3]. The formal resemblance between electrons in crystals and Bose-Einstein condensates (BECs) in optical lattices has inspired an extensive interest to explore the BOs in optical lattice system. The successful observations of BOs have been reported for atoms in interacting BECs [4, 5, 6, 7] and for ensembles of noninteracting quantum-degenerate fermions [8] in tilted optical lattices.

However, the perfect BOs can only be available in the ideal case where there is no interactions among the atoms of BECs. Practically, due to the intrinsically weak interactions of atoms, the momentum distribution of the BECs will show a rapid broadening, which causes the atoms of BECs to lose their phase coherence, i.e. the dephasing effect [9, 10]. Consequently, the BOs in BECs cannot persist on for a long time. In the framework of the mean-field treatment, the motion of the BECs can be well described by the so-called Gross-Pitaevskii equation (GPE) with a nonlinear term. It is believed that such nonlinearity induced by the atom-atom interactions generally leads to a breakdown of the BOs, as recently studied experimentally [11] and theoretically [12, 13, 14]. Therefore, a natural question is: Is it possible to prolong or even stabilize the BOs by some active control ways?

Addressing this question, some progress, especially in single-component BECs, has been made. It has been found that a long-living BO can be induced by properly designing the spatial dependence of the scattering length [15] and the configuration of the optical lattices [16]. Gustavsson *et al.* [11] showed the experimental evidence that the dephasing time of the BOs can be much enhanced by decreasing the interaction strengths via the Feshbach resonance [17]. However, in many situations, such finite enhancement to the dephasing time of BOs is not enough, and one is desired to preserve the BOs forever. Recently, Gaul *et al.* have reported that a persistent BO of single-component BECs can be obtained by modulating interaction harmonically in time with suitable frequency and phase [10, 18], which can be easily realized by means of Feshbach resonance.

So far, most of the studies of the stabilization control to BO of BECs in optical lattice are based on the single-component BEC case. Compared with single-component BEC, the two-component BEC system may exhibit more novel physical effects due to the condensate mixtures [19, 20, 21, 22]. In this system, both intracomponent and intercomponent atom-atom interactions take effects upon the nonlinear behavior of the BECs. The two-component BECs can be experimentally realized by spin-dependent optical lattices, which hold the BECs with the two components composed of two distinct

hyperfine states of the same atomic species [23].

In the present paper, we study systematically the modulation stabilization of the BOs of two-component BECs in optical lattices. We mainly use two control ways, one is by modulating periodically the interactions via Feshbach resonance; the other is by tuning statically the parameters of the optical lattices. We will show that the stable BOs can be obtained when the interactions are modulated synchronously and harmonically in time with suitable frequencies. Moreover, if the intercomponent and the intracomponent interactions have inverse signs, the long-living BOs and even the revival of the BOs can be achieved via only tuning the relative separation between lattices.

The paper is organized as follows. In Sec. II, we discuss the methods and formalism used in this work. In Sec. III, we explore quantum manipulation of BOs from two aspects according to the signs of the intracomponent atom-atom interactions. Finally, a summary is given in Sec. IV.

## 2. Model and formulation

### 2.1. Gross-Pitaevskii equations for the two-component BECs in an optical lattice

We consider two-component BECs which are composed of bosonic atoms of the same isotope but having different internal spin states, e.g.  $^{87}\text{Rb}$  atoms in hyperfine states  $|F = 2, m_F = 2\rangle$  and  $|F = 1, m_F = -1\rangle$  [24]. The BECs are trapped in spin-dependent optical lattices. The dynamics of the system is governed by the coupled GPEs under the mean field approximation,

$$i\hbar\partial_t\Phi_i = \left[-\frac{\hbar^2}{2M}\nabla^2 + V_i + \sum_{j=1}^2 g_{ij}(t)|\Phi_j|^2\right]\Phi_i, \quad (1)$$

where  $\Phi_i$  ( $i = 1, 2$ ) is the macroscopic condensate order parameter of the  $i$ -th component with identical mass  $M$ . The time-dependent interaction coefficients are given by  $g_{ij}(t) = 4\pi\hbar^2 a_{ij}(t)/M$  with  $a_{ij}(t)$  being the  $s$ -wave scattering length which can be controlled via the Feshbach resonance induced by the modulated magnetic field. The external potential felt by the  $i$ -th component can be decomposed into  $V_i = V_c + V_{Li}$ , where  $V_c = fz$  is a linear potential induced by a constant force  $f$  and  $V_{Li}$  is trapping potential of the optical lattice. The additional weak potential  $V_c$  tilts the optical potentials and drives coherent oscillations [5]. The trapping potentials for different components can be explicitly written as  $V_{L1} = U_p \sin^2(k_L z + \frac{\theta}{2})$  and  $V_{L2} = U_p \sin^2(k_L z - \frac{\theta}{2})$ , where  $U_p$  is the depth of the 1D lattice potentials,  $k_L$  is the wave vector of the lasers used to construct the optical lattice, and  $\theta$  is the polarization angle of the two counterpropagating laser beams to form the standing wave configuration of the optical lattice [20, 23]. By changing  $\theta$ , one can also control the separation between the two potentials.

When the linear field is too weak to induce Landau-Zener tunneling [25, 26], BO can be described by an adiabatic evolution of the BECs in the lowest lattice band. In collective coordinates [9], the condensate order parameter  $\Phi_i(r, t)$  can be expanded as a

linear combination of the wave packets localized at the individual lattice sites, i.e. the Wannier wave functions  $\phi_{n_i}(r)$ , as

$$\Phi_i(r, t) = \sqrt{N_i} \sum_{n_i} \psi_{i,n_i}(t) \phi_{n_i}(r), \quad (2)$$

where  $N_i$  is the total number of particles of the  $i$ -th component and the Wannier wave function  $\phi_{n_i}$  satisfies the orthogonality condition  $\int dr \phi_{n_i} \phi_{n_i \pm 1} = 0$ , and the normalization condition  $\int dr \phi_{n_i}^2 = 1$ .  $\psi_{i,n_i} = \sqrt{N_{i,n_i}(t)/N_i} e^{i\theta_{i,n_i}(t)}$ , where  $N_{i,n_i}(t)$  and  $\theta_{i,n_i}(t)$  are the number of particles and phase, respectively, is the amplitude of the  $i$ -th component trapped in the site  $n_i$ . In the following, we assume that the two components have the same total number of particles, i.e.,  $N_1 = N_2$ . Substituting Eq. (2) into Eqs. (1), we can discretize Eqs. (1) into a set of coupled nonlinear equations with respect to different lattice site  $n_i$ ,

$$\begin{aligned} i\dot{\psi}_{i,n_i} = & -\frac{\psi_{i,n_i-1} + \psi_{i,n_i+1}}{2} + [\epsilon_{i,n_i} + \Lambda_{ii}(t)|\psi_{i,n_i}|^2 \\ & + \Lambda_{ij}(t)(\eta_\tau |\psi_{j,n_i+\tau}|^2 + \eta_{\tau-1} |\psi_{j,n_i+\tau-1}|^2)] \psi_{i,n_i}, \end{aligned} \quad (3)$$

where the overdot denotes the time derivative and  $i \neq j$  labels the two different components of BECs.  $\epsilon_{i,n_i} = \frac{1}{2J} \int dr [\frac{(\hbar \nabla \phi_{n_i})^2}{2M} + V_i \phi_{n_i}^2]$  with  $J = - \int dr [\frac{\hbar^2}{2M} \nabla \phi_{n_i} \nabla \phi_{n_i+1} + \phi_{n_i} V_i \phi_{n_i+1}]$  being the tunnel parameter.  $\Lambda_{ii} = \frac{g_{ii}(t)N_i}{2J} \int dr \phi_{n_i}^4$  describes the intracomponent atom-atom interaction strength.  $\Lambda_{ij} = \frac{g_{ij}(t)N_j}{2J}$  multiplying with  $\eta_\tau = \int dr \phi_{n_i}^2 \phi_{n_i+\tau}^2$  and  $\eta_{\tau-1} = \int dr \phi_{n_i}^2 \phi_{n_i+\tau-1}^2$  describe the intercomponent atom-atom interaction strengths, where  $\eta_\tau$  and  $\eta_{\tau-1}$  stem from the overlaps of the wave functions of the two components.  $\tau$  (in the lattice unit) is determined by the relative separation between the two nearest neighboring spin-dependent potentials, which can be controlled by the polarization angle  $\theta$ . It is noted that not only all the interactions are time-dependent but also intercomponent nonlinear interactions depend on  $\eta_\tau$  and  $\eta_{\tau-1}$ . In Eq. (3) the time has been rescaled to be dimensionless as  $t \rightarrow \hbar t/2J$ .

In 1D optical potentials, we can denote the Wannier wave function as  $\phi_{n_i}(r) = \phi(x, y) \phi_{n_i}(z)$ . The transverse wave function can be expressed as a 2D Gaussian profile  $\phi(x, y) = \phi_0(x) \phi_0(y)$ , where  $\phi_0(\alpha) = \frac{1}{\sqrt{4\pi\sigma_\alpha}} \exp(-\frac{\alpha^2}{2\sigma_\alpha^2})$  with  $\sigma_\alpha$  being the Gaussian widths in the  $\alpha = x, y$  directions. The wave function along the direction of the optical lattice can be denoted as  $\phi_{n_i}(z) = \phi_0(z - n_i d)$  with  $d = \pi/k_L$  being the lattice constant. Based on the variational ansatz for  $\phi_{n_i}(z)$ , a minimum energy can be obtained when the width of  $\phi_{n_i}(z)$  equals to  $\sigma_z = \frac{d}{\pi} \sqrt{4U_p/E_{rec}}$ , where  $E_{rec}$  is the recoil energy [25]. Under this consideration, the parameters in Eq. (3) can be determined explicitly as

$$\begin{aligned} \epsilon_{1,n_1} &= \omega n_1 + \frac{\theta^2 U_p}{8}, \quad \epsilon_{2,n_2} = \omega n_2 - \frac{\theta^2 U_p}{8}, \\ \Lambda_{ii}(t) &= \frac{1}{2J} \frac{g_{ii}(t)N_i}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z}, \quad \eta_\tau = \exp(-\frac{\tau^2 d^2}{2\sigma_z^2}), \\ J &= \exp(\frac{d^2}{4\sigma_z^2}) \left\{ \frac{\hbar^2}{2M} \frac{d^2 - 2\sigma_z^2}{4\sigma_z^4} - \frac{U_p}{2} [1 + \exp(\frac{-\pi^2 \sigma_z^2}{d^2})] \right\}, \end{aligned} \quad (4)$$

where  $\omega = \frac{fd}{2J}$  and  $f$  corresponds to the weak atomic gravity. From Eq. (3) and the canonical equation  $\dot{\psi}_i = \frac{\partial \mathcal{H}}{\partial (i\psi_i^*)}$ , the Hamiltonian functions can be obtained

$$\begin{aligned} \mathcal{H}_i = \sum_{n_i} \{ & -\frac{\psi_{i,n_i}\psi_{i,n_i+1}^* + \psi_{i,n_i}^*\psi_{i,n_i+1}}{2} + [\epsilon_{i,n_i} \\ & + \frac{\Lambda_{ii}(t)}{2}|\psi_{i,n_i}|^2 + \Lambda_{ij}(t)(\eta_\tau|\psi_{j,n_i+\tau}|^2 \\ & + \eta_{\tau-1}|\psi_{j,n_i+\tau-1}|^2)]|\psi_{i,n_i}|^2\}, \end{aligned} \quad (5)$$

both the Hamiltonian  $\mathcal{H}_i$  and the norm  $\sum_{n_i} |\psi_{i,n_i}|^2 = N_i$  are conserved.

## 2.2. The dynamics of wave packet: Bloch oscillations

In order to analyze how the interactions affect the BOs of the two-component BECs, we parameterize the Gaussian profile wave packet for  $i$ -th component as [9]

$$\psi_{i,n_i} = \sqrt{K_i} \exp\left[-\frac{(n_i - \xi_i)^2}{\gamma_i^2} + ip_i(n_i - \xi_i) + i\frac{\delta_i}{2}(n_i - \xi_i)^2\right]. \quad (6)$$

where  $K_i = \sqrt{\frac{2}{\pi\gamma_i^2}}$  is a normalization factor. The Gaussian wave packet is parameterized by four types of quantities: the center-of-mass position  $\xi_i(t)$ , the width of the wave packet described by  $\gamma_i(t)$ , the linear phase  $p_i(t)$  describing the group velocity of the wave packet, and the quadratic phase  $\delta_i(t)$  over the wave packet. The latter phase allows us, on the one hand, to describe the linear evolution of the wave packet for which the quadratic dispersion in momentum space directly translates into a quadratic phase in real space. On the other hand, the nonlinearity due to the atom-atom interactions also leads to a quadratic phase since the density near the Gaussian maximum is quadratic [1]. Such Gaussian profile wave packet was used to explain successfully the BO in Anderson-Kasevich experiment [5].

The dynamical evolution of the wave packet can be obtained by a variational principle from the Lagrangian  $\mathcal{L}_i = i\sum_{n_i} \dot{\psi}_{i,n_i}\psi_{i,n_i}^* - \mathcal{H}_i$ . After some algebra, the Lagrangian can be achieved

$$\begin{aligned} \mathcal{L}_i = p_i\dot{\xi}_i - \frac{\gamma_i^2\dot{\delta}_i}{8} + e^{-\chi_i} \cos p_i - \frac{\Lambda_{ii}(t)}{2\sqrt{\pi}\gamma_i} - v_i \\ - \Lambda_{ij}(t)\frac{\kappa}{\sqrt{\pi}}[\eta_\tau e^{-\mu_\tau} + \eta_{\tau-1}e^{-\mu_{\tau-1}}], \end{aligned} \quad (7)$$

where  $\chi_i = \frac{1}{2\gamma_i^2} + \frac{\gamma_i^2\delta_i^2}{8}$ ,  $v_i = K_i \int dn_i \epsilon_{i,n_i} \exp[\frac{-2(n_i - \xi_i)^2}{\gamma_i^2}]$ ,  $\kappa = \frac{\sqrt{2}}{\gamma}$  with  $\gamma^2 = \gamma_1^2 + \gamma_2^2$ , and  $\mu_\tau = \kappa^2 \xi_\tau^2$  with  $\xi_\tau = \xi_i - \xi_j + \tau$ . It is noted that in our calculation the summation over  $n_i$  has been replaced by integration when the widths  $\gamma_i$  are not too small [9, 22]. The equations of motion of the collective coordinates can be obtained from the Euler-Lagrange equations

$$\dot{\xi}_i = e^{-\chi_i} \sin p_i, \quad (8)$$

$$\dot{\gamma}_i = \gamma_i \delta_i e^{-\chi_i} \cos p_i, \quad (9)$$

$$\dot{p}_i = \frac{2\kappa^3 \Lambda_{ij}(t)}{\sqrt{\pi}} [\eta_\tau \xi_\tau e^{-\mu_\tau} + \eta_{\tau-1} \xi_{\tau-1} e^{-\mu_{\tau-1}}] - \omega, \quad (10)$$

$$\dot{\delta}_i = \left(\frac{4}{\gamma_i^4} - \delta_i^2\right) e^{-\chi_i} \cos p_i + \frac{2\Lambda_{ii}(t)}{\sqrt{\pi}\gamma_i^3} + \frac{\kappa^5 z \Lambda_{ij}(t)}{\sqrt{\pi}}, \quad (11)$$

where  $z = \eta_\tau(\gamma^2 - 4\xi_\tau^2)e^{-\mu_\tau} + \eta_{\tau-1}(\gamma^2 - 4\xi_{\tau-1}^2)e^{-\mu_{\tau-1}}$ .

To highlight the essential physics, from the coupled Eqs. (8)-(11), the equation of motion of the center of the wave packet can be recast into

$$\ddot{\xi}_i + \alpha_i \dot{\xi}_i = \beta_i, \quad (12)$$

where

$$\alpha_i = \frac{\delta_i \Lambda_{ii}(t)}{2\sqrt{\pi}\gamma_i} + \frac{\kappa^5 \gamma_i^2 \delta_i \Lambda_{ij}(t) z}{4\sqrt{\pi}}, \quad (13)$$

$$\beta_i = \dot{p}_i e^{-\chi_i} \cos p_i. \quad (14)$$

It is noted that Eq. (12) can recover the equation of motion of the wave-packet center for single component BEC under the condition  $\Lambda_{ij} = 0$  as [9]

$$\ddot{\xi}_i + \alpha_i \dot{\xi}_i + \omega^2 \xi_i = \frac{\omega \Lambda_{ii}}{2\sqrt{\pi}} [\gamma_i^{-1}(0) - \gamma_i^{-1}(t)], \quad (15)$$

which is a standard equation of motion for a harmonic oscillation with an effective damping rate  $\alpha(t)$ . Under the ideal condition  $\Lambda_{ii} = 0$ ,  $\alpha(t) = 0$  and the system undergoes a perfect oscillation, which is the perfect BOs with the frequency  $\omega$  as a result of the driven field. The damping of the BOs is caused by the intracomponent interactions  $\Lambda_{ii}$ , which means that the nonlinearities inevitably lead to the breakdowns of the BOs. In the situation of two-component BECs, the intercomponent interaction  $\Lambda_{ij}$  will add another nonlinearity to each individual component of the BECs, which gives an additional contribution to the damping rate, as shown in Eq. (13). Besides the damping rate, the intercomponent interaction also exerts an effective driven force to BOs. Explicitly, under the condition that  $\Lambda_{ij}$  is small compared with the constant force  $f$  felt by the BECs, one can rewritten Eq. (12) as

$$\ddot{\xi}_i + \alpha_i \dot{\xi}_i + \tilde{\omega}^2 \xi_i = \eta_i(\Lambda_{ii}, \Lambda_{ij}), \quad (16)$$

which shows that the perfect BOs of the system are distorted by the cooperative action of the effective damping rate  $\alpha(t)$  and the effective driven force  $\eta_i(\Lambda_{ii}, \Lambda_{ij})$ . It is noted that the frequency  $\tilde{\omega}$  of the BOs, which corresponds to the inverse of the right-hand side of Eq. (10), for the two-component case is slightly detuned from  $\omega$  by the intercomponent interactions  $\Lambda_{ij}$ . The explicit analysis of the BOs governed by  $\alpha(t)$  and  $\eta_i(\Lambda_{ii}, \Lambda_{ij})$  will be shown by the quantitative calculations with the experimentally adjustable parameters in the next section.

### 3. Quantum manipulation of Bloch oscillation in the optical lattice

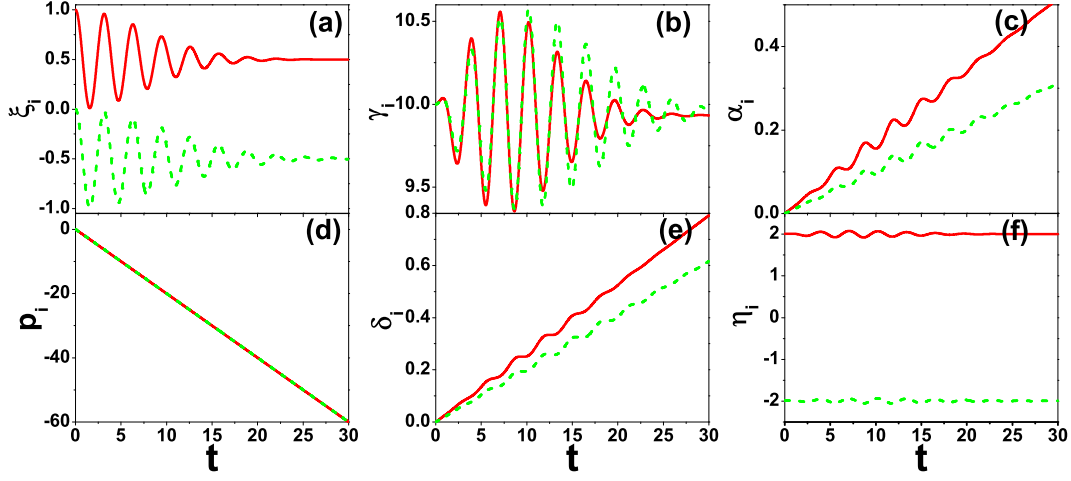
From the above analytical results, we can see that the nonlinearities contributed from both of the intercomponent and intracomponent interactions generally lead to the

breakdowns of the BOs of the BECs in the optical lattices. Therefore, the interaction coefficients  $g_{ij}$  of such nonlinearities, which are essentially determined by the  $s$ -wave scattering lengths  $a_{ij}$ , have profound impact on the dynamics of two-component BECs in the optical lattices. The magnitude and sign of these parameters sensitively influence dynamical behaviors of this ultracold boson system. In cold-atom experiments, Feshbach resonance is a quite effective mechanism that can be used to modulate  $g_{ij}$ . Inspired by a recent experimental investigation of the ultracold molecule production via a sinusoidal magnetic field modulation to the interaction coefficient around the Feshbach resonance [27], we intend to explore the possibility to stabilize the BOs via such periodic modulations to the interaction coefficients  $g_{ij}$  in the following. Besides the magnetic field induced Feshbach resonance, another way to modulate the dynamics of the BECs in our system is via adjusting the configuration of the optical lattices. The separation between the spin-dependent potentials felt by the two components of BECs, which can be adjusted by the polarization angle  $\theta$  of the lasers, essentially determines the intercomponent interactions of the BECs. We also examine the influence of the separation on the dynamics of BOs. The combined effect of the Feshbach resonance and a periodic external potential has been widely studied [28, 29, 30].

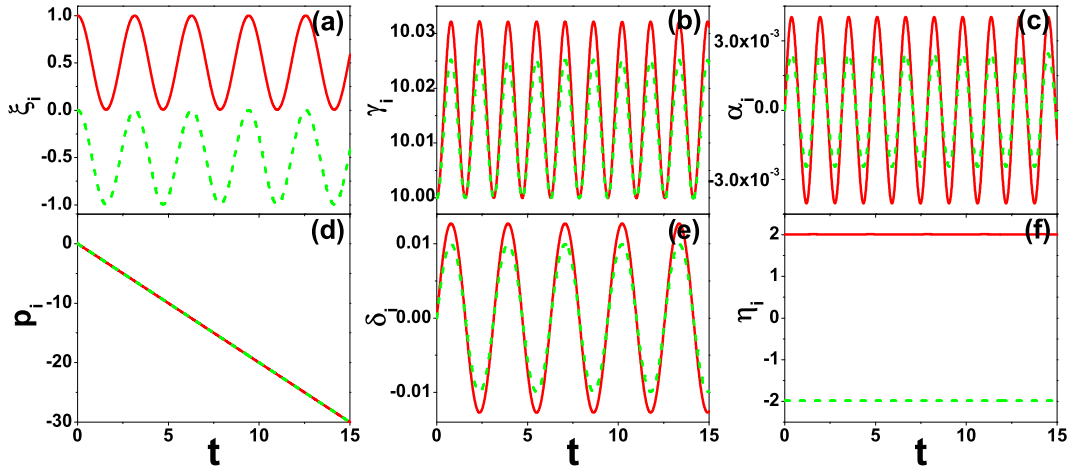
### 3.1. $a_{ii}(0) > 0, a_{ij}(0) > 0$

In this case, both of the intercomponent and intracomponent interactions are repulsive. Without loss of generality, we assume that the intercomponent interactions  $\Lambda_{ij}$  relate to the intracomponent interactions  $\Lambda_{ii}$  as:  $\Lambda_{12} = \Lambda_{21} = \sqrt{\Lambda_{11}\Lambda_{22}}$  in our numerical simulations.

To see the effect of nonlinearities on the BOs, we plot in Fig. 1 the attenuations of BOs without modulations. From the time-dependent behaviors of the wave-packet centers  $\xi_i$  [Fig. 1(a)] we can see clearly the breakdown of BOs with time evolution. Such damping oscillations are manifested by the behaviors of the effective damping rates  $\alpha_i$  [Fig. 1(c)] and the driven force  $\eta_i$  [Fig. 1(f)], where  $\alpha_i$  are always positive and increase with time, and  $\eta_i$  tend to constants after the same cycles as  $\xi_i$ . Compared with the case for the single-component BECs [9], the presence of the intercomponent interactions, sharing the same sign with intracomponent ones here, play the role as an additional nonlinearities and speed up the collapses in our two-component situation. So, the stronger the intercomponent interactions are, the faster the BOs damp. Fig. 1(d) shows that  $p_i$ , the associated momenta of  $\xi_i$ , increase linearly with time. This can be understood from the analysis of Eq. (10). The first term of the right hand of Eq. (10), which is contributed from the intercomponent interactions, is much smaller than the second term, which is contributed from the linear potential. Consequently, the time-dependent behaviors of  $p_i$  are dominated by the second term, i.e.  $p_i(t) \approx p_i(0) - \omega t$ . Fig. 1(b) shows that the widths  $\gamma_i$  undergo breathing oscillations and soon approach constants. The associated momenta  $\delta_i$  are also divergent, as shown in Fig. 1(e). All these time-dependent behaviors indicate that the system is set into a macroscopical quantum



**Figure 1.** (Color online) Attenuations of BOs without modulating the interactions. Solid and dashed lines indicate two individual components with the intracomponent interactions as  $\Lambda_{11} = 20$  and  $\Lambda_{22} = 15$ , respectively. The other parameters are chosen as:  $U_p = 16E_{rec}$ ,  $\omega = 2$ , and  $\tau = 0.5$ . The initial conditions are set as:  $p_1(0) = p_2(0) = 0$ ,  $\delta_1(0) = \delta_2(0) = 0$ ,  $\xi_1(0) = 1$ ,  $\xi_2(0) = 0$ ,  $\gamma_1(0) = \gamma_2(0) = 10$ .



**Figure 2.** (Color online) The stabilization of BOs by modulating the interactions. All interactions are modulated by  $\cos(\omega t)$ . The same parameters and notation are used as Fig. 1.



self-trapping mode [31, 32, 33] due to the nonlinearities from the intercomponent and intracomponent interactions.

Now we focus on how to stabilize the BOs in our system. We mainly use the way by modulating periodically the interactions via a magnetic field.

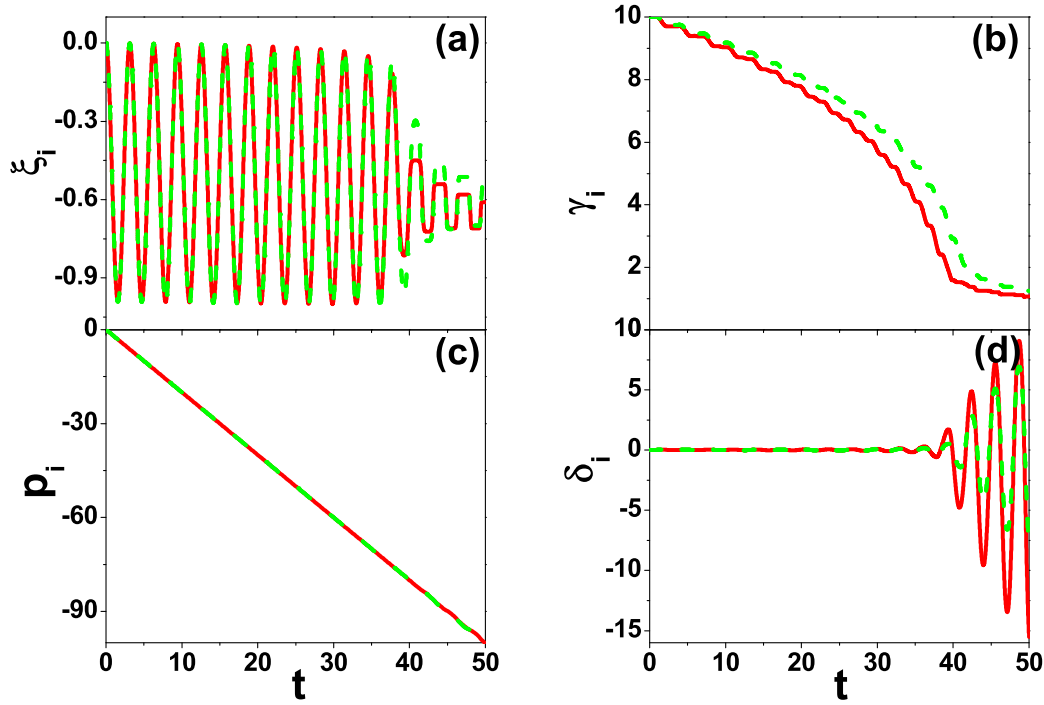
In Fig. 2 we plot the time-dependent behaviors of the wave-packet variables under a  $\cos\omega t$ , where  $\omega$  is the frequency of BOs, modulation to all the interactions. We can see that the damping of the BOs can be fully stabilized by such modulation and perfect oscillations is thus obtained. Such persistent phenomena can be also explained straightforwardly by studying the time-dependence of the damping coefficients  $\alpha_i$  [Fig. 2(c)] and the effective driven forces  $\eta_i$  [Fig. 2(f)]. In contrast to the positivity in the full evolution range in Fig. 1(c),  $\alpha_i$  in this case exhibit periodic oscillations between positive and negative values with definite amplitudes, which characterizes well the lossless BOs of  $\xi_i$ . It means that the effective damping coefficients with alternate signs inspire the system itself to guarantee the stabilizations of the BOs. Besides, we find that the driven forces in Fig. 2(f) are replaced by constant values completely after such modulation. It has been proven that there is a family of stable solutions in terms of collective coordinates in single-component BEC system when all the interactions are modulated by  $\cos(\omega t)$  [10]. In fact, in two-component ones, the coupled terms in the Eqs. (1) can be regarded as additional nonlinearities, which possess the same time dependence as intracomponent interactions so long as modulating all interactions harmonically in time with the same suitable frequencies. As viewed from the mean field, the two components can be reduced into two independent single ones. So it is understandable that such modulation stabilization also presents in our two-component BECs system.

It is noted that the effect of the modulation to the BOs is sensitively dependent of the forms of modulating field we used. In Fig. 3, we plot the numerical simulation when all the interactions are modulated by a  $\sin(\omega t)$  field. It is found that the damping of the BOs manifested by  $\xi_i$  is not suppressed and the BOs is destroyed in several rounds of oscillation. The wave-packet widths  $\gamma_i$  reduce their amplitudes quickly. So the modulation takes no effect in this case.

In the present case, the intercomponent interactions share the same sign as the intracomponent ones, so the tuning of the relative separation  $\tau$  does not have constructive action to suppress the damping of BOs. However, the things are changed dramatically when the intercomponent interactions have opposite sign to intracomponent ones, as discussed in the following.

### 3.2. $a_{ii}(0) > 0, a_{ij}(0) < 0$

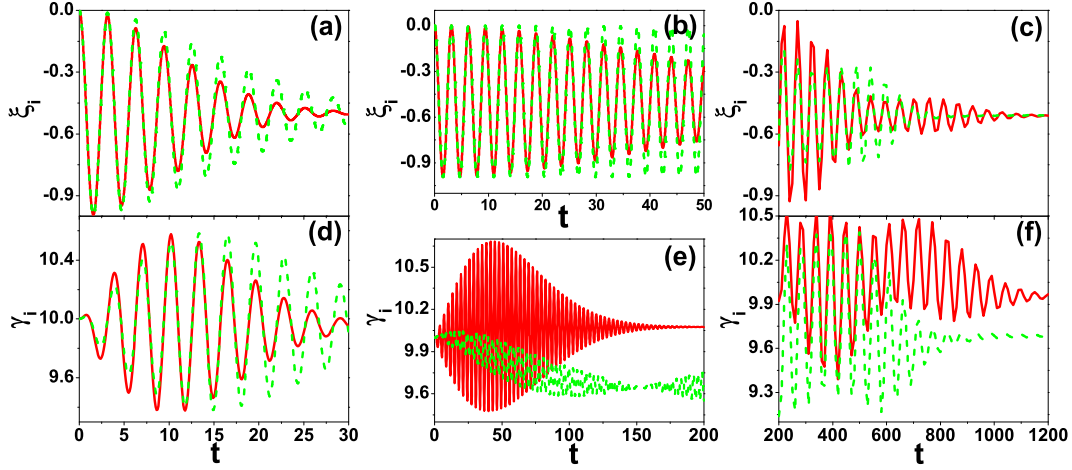
In this case, the original intracomponent interactions are repulsive, while the original intracomponent ones are attractive. Different to the above case, there are many interesting effects induced from the competition between such two kinds of atom-atom interactions. For example, the stability of static solitonic excitations in two-component BECs have been analyzed within the Gross-Pitaevskii approximation [34].



**Figure 3.** (Color online) The attenuations of the BOs with modulating interactions by  $\sin(\omega t)$ .  $\tau = 0.1$  and the same other parameters and notation as Fig. 1.

For convenience, we assume the intercomponent atom-atom interactions to be  $\Lambda_{12} = \Lambda_{21} = -\sqrt{\Lambda_{11}\Lambda_{22}}$  in our numerical simulations.

As analyzed in above case, the BOs are destroyed by the nonlinearities. From this point, the dynamical behaviors of the wave packet in the present case show no difference to the above one. However, we can prolong the coherent time of the BOs dramatically by tuning the relative separation  $\tau$  of the potentials felt by the two components in the present case. To confirm this, we plot the time evolutions of the wave-packet centers  $\xi_i$  and widths  $\gamma_i$  in Fig. 4 for different relative separations. A large  $\tau$  means a large distances between the nearest neighbors of the Wannier wave functions, which in turn induces a small intercomponent interaction rate  $\eta_\tau$ . Fig. 4(a,d) show the breakdown of the BOs when the intercomponent interactions are small for a large  $\tau$ . With the decrease of  $\tau$ , the intercomponent interactions get stronger. The damping of BOs are obviously slowed down [Fig. 4(b,e)]. Especially, it is noticed that  $\gamma_i$  shows revival at about  $t = 150$ , as the dashed line in Fig. 4(e). It provides a valuable guidance that the dynamics of the system would show revival in this case. If the relative separation  $\tau$  is further reduced so that the two lattices are extremely close, the BOs show obvious revival [Fig. 4(c,f)]. This phenomenon is caused by the competition between the intercomponent and intracomponent interactions. Because the intracomponent interactions have inverse sign



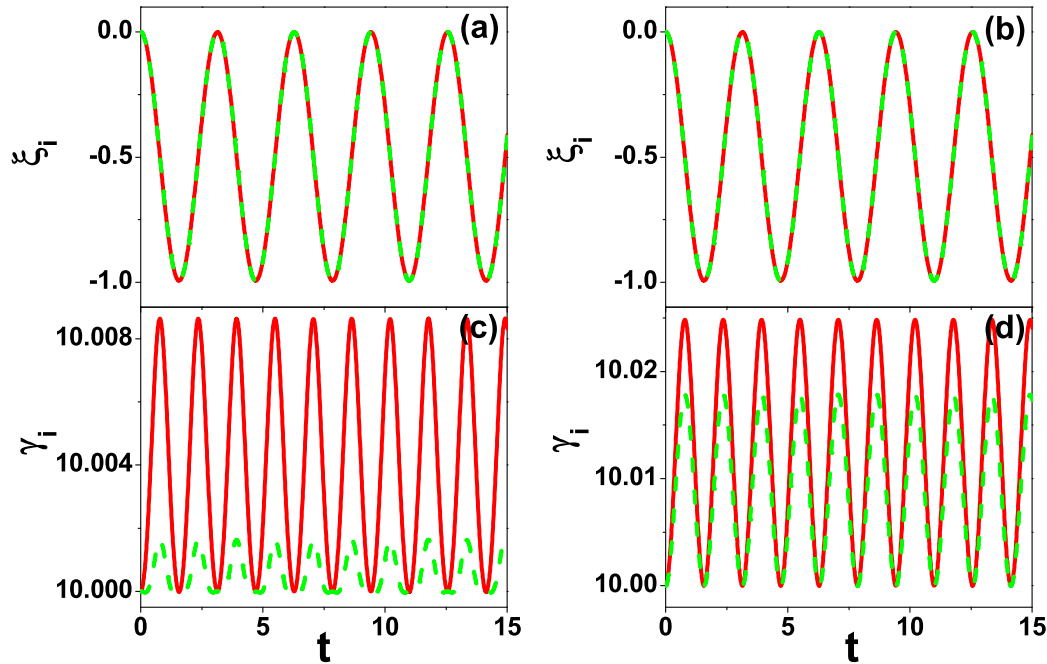
**Figure 4.** (Color online) Attenuations of BOs without modulating interactions for different potential separations  $\tau$ . Solid and dashed lines indicate two individual components with the intracomponent interactions as  $\Lambda_{11} = 20$  and  $\Lambda_{22} = 18$ , respectively. The other parameters are chosen as:  $U_p = 16E_{rec}$ ,  $\omega = 2$ , and  $\tau = 0.5$  for (a, d),  $\tau = 0.1$  for (b, e), and  $\tau = 0.05$  for (c, f). The initial conditions are set as:  $p_1(0) = p_2(0) = 0$ ,  $\delta_1(0) = \delta_2(0) = 0$ ,  $\xi_1(0) = \xi_2(0) = 0$ ,  $\gamma_1(0) = \gamma_2(0) = 10$ .

with the intercomponent ones, the nonlinearities contributed from the intracomponent interactions are partially counteracted by the intercomponent ones. For the full overlap at  $\tau = 0$ , the two components perfectly mix together and attenuations of BOs reappear. To sum up, the coherent time of the BOs can be much enhanced by only tuning the relative separation  $\tau$  of the optical lattices.

However, in many situations, such finite enhancement to the dephasing time of BOs is not enough, and one is desired to preserve the BOs forever. This actually can also be achieved by modulating the interactions by a suitable time-dependent field. Fig. 5 shows that the BOs are entirely stabilized by modulating interactions harmonically in time with the same frequency as the one of the BOs. Such stabilization is independent of the magnitudes of the nonlinearities, so the behaviors for different  $\tau$  under the modulation are same, as shown in Fig. 5(a,b).

#### 4. Conclusions

In summary, we have studied analytically and numerically the dynamical behaviors of the BOs for two-component BECs trapped in combined potentials consisting of linear potentials and spin-dependent optical lattices. We found that the damped BOs can be stabilized when all the atom-atom interactions are modulated synchronously and harmonically in time with Bloch frequency. Moreover, if the intercomponent and the intracomponent interactions have inverse signs, it has been shown that the dephasing



**Figure 5.** (Color online) The stabilization of BOs for different relative separations  $\tau$  under the  $\cos(\omega t)$  modulation.  $\tau = 0.1$  for (a, c) and  $\tau = 0.5$  (b, d). The same other parameters and notation as Fig. 4.

time of BOs can be much enhanced by decreasing the relative separation of the two potentials felt by the two components. Our results provide a valuable guidance for achieving long-lived BOs in the two-component BEC system by the Feshbach resonances and manipulating the configuration of the optical lattices.

### Acknowledgements

This work was supported by the Fundamental Research Fund for Physics and Mathematics of Lanzhou University (Grant LZULL200806). JHA thanks the support by the Fundamental Research Funds for the Central Universities under Grant No. lzujbky-2010-72 and the Gansu Provincial NSF of China under Grant No. 0803RJZA095.

### References

- [1] Morsch O and Oberthaler M 2006 *Rev. Mod. Phys.* 78 179.
- [2] Bloch I 2005 *Nature Phys.* 1 23.
- [3] Bloch F 1929 *Z. Phys* 52 555.
- [4] Dahan M B, Peik E, Reichel J, Castin Y and Salomon C 1996 *Phys. Rev. Lett.* 76 4508.

- [5] Anderson B P and Kasevich M A 1998 *Science* 282 1686.
- [6] Morsch O, Müller J H, Cristiani M, Ciampini D and Arimondo E 2001 *Phys. Rev. Lett.* 87 140402.
- [7] Salger T, Ritt G, Geckeler C, Kling S and Weitz M 2009 *Phys. Rev. A* 79 011605(R).
- [8] Roati G, de Mirandes E, Ferlaino F, Ott H, Modugno G and Inguscio M 2004 *Phys. Rev. Lett.* 92 230402.
- [9] Trombettoni A and Smerzi A 2001 *Phys. Rev. Lett.* 86 2353.
- [10] Gaul C, Lima R P A, Díaz E, Müller C A and Domínguez-Adame F 2009 *Phys. Rev. Lett.* 102 255303.
- [11] Gustavsson M, Haller E, Mark M J, Danzl J G, Rojas-Kopeinig G and Nägerl H C 2008 *Phys. Rev. Lett.* 100 080404.
- [12] Gangardt D M and Kamenev A 2009 *Phys. Rev. Lett.* 102 070402.
- [13] Krimer D O, Khomeriki R and Flach S 2009 *Phys. Rev. E* 80 036201.
- [14] Kolovsky A R, Gómez E A and Korsch H J 2010 *Phys. Rev. A* 81 025603.
- [15] Salerno M, Konotop V V and Bludov Y V 2008 *Phys. Rev. Lett.* 101 030405.
- [16] Walter S, Schneble D and Durst A C 2010 *Phys. Rev. A* 81 033623.
- [17] Donley E A, Claussen N R, Cornish S L, Roberts J L, Cornell E A and Wieman C E 2001 *Nature* 412 295.
- [18] Díaz E, Gaul C, Lima R P A, Domínguez-Adame F and Müller C A 2010 *Phys. Rev. A* 81 051607(R).
- [19] Pu H and Bigelow N P 1998 *Phys. Rev. Lett.* 80 1130.
- [20] Ostrovskaya E A and Kivshar Y S 2004 *Phys. Rev. Lett.* 92 180405 .
- [21] Ma X, Xia L, Yang F, Zhou X, Wang Y, Guo H and Chen X 2006 *Phys. Rev. A* 73 013624.
- [22] Wang J-J, Zhang A-X, Zhang K-Z, Ma J and Xue J-K 2010 *Phys. Rev. A* 81 033607.
- [23] Mandel O, Greiner M, Widera A, Rom T, Hänsch T W and Bloch I 2003 *Phys. Rev. Lett.* 91 010407.
- [24] Schmaljohann H, Erhard M, Kronjäger J, Kottke M, van Staa S, Cacciapuoti L, Arlt J J, Bongs K and Sengstock K 2004 *Phys. Rev. Lett.* 92 040402.
- [25] Cristiani M, Morsch O, Müller J H, Ciampini D and Arimondo E 2002 *Phys. Rev. A* 65 063612.
- [26] Witthaut D, Werder M, Mossmann S and Korsch H J 2005 *Phys. Rev. E* 71 036625.
- [27] Thompson S T, Hodby E and Wieman C E 2005 *Phys. Rev. Lett.* 95 190404.
- [28] Abdullaev F K, Tsoy E N, Malomed B A and Kraenkel R A 2003 *Phys. Rev. A* 68 053606.
- [29] Abdullaev F K, Baizakov B B, Darmanyan S A, Konotop V V and Salerno M 2001 *Phys. Rev. A* 64 043606.
- [30] Brazhnyi V A and Konotop V V 2005 *Phys. Rev. A* 72 033615.
- [31] Smerzi A and Raghavan S 2000 *Phys. Rev. A* 61 063601.
- [32] Milburn G J, Corney J, Wright E M and Walls D F 1997 *Phys. Rev. A* 55 4318.
- [33] Alexander T J, Ostrovskaya E A and Kivshar Y S 2006 *Phys. Rev. Lett.* 96 040401.
- [34] Schumayer D and Apagyí B 2004 *Phys. Rev. A* 69 043620.